- **Laplace Transform**: $\mathcal{L}[f(t)] = F(s) = \int_0^\infty f(t)e^{-st}dt$
  - Continuous version of power series
  - $\mathcal{L}$ transforms a function of t into a function of s.
  - $\mathcal{L}$ is a linear transform as integration is linear.
    - $\mathcal{L}[c_1 f(t) + c_2 g(t)] = c_1\mathcal{L}[f(t)] + c_2\mathcal{L}[g(t)]$
  - Why are Laplace transforms useful?
    - Solve differential equations
    - Circuit analysis (e.g. finding the complex impedance of a capacitor)
  - Laplace transforms only exist for functions of exponential type (i.e. $f(t) \le Ce^{kt}$)
    - Example: $f(t) = \frac{1}{t}$ and $f(t) = e^{t^2}$ does not have a Laplace transform
- How to solve a differential equation using Laplace transforms:
  - 1. Take the Laplace transform of both sides.
  - 2. Solve for $F(s)$.
  - 3. Take the **inverse Laplace transform** $\mathcal{L}^{-1}$ of both sides to find $f(t)$.
    - If $\mathcal{L}[f(t)] = F(s)$, then $\mathcal{L}^{-1}[F(s)] = f(t)$
    - Remembering how to do partial fractions will be very helpful!
  - Why would this method ever be preferred over previous methods?
    - This method is often shorter depending on the type of function you have, such as periodic functions or certain discontinuous functions
- Common Laplace Transforms (see next page for a more complete list)
  - $\mathcal{L}[1] = \frac{1}{s},\ s > 0$
  - $\mathcal{L}[e^{at}] = \frac{1}{s-a},\ s > a$
  - $\mathcal{L}[e^{at} f(t)] = F(s-a)$ Exponential s-shift
  - $\mathcal{L}[\cos\omega t] = \frac{s}{s^2+\omega^2}$ $\mathcal{L}[\sin\omega t] = \frac{\omega}{s^2+\omega^2}$
  - $\mathcal{L}[\cosh\omega t] = \frac{s}{s^2-\omega^2}$ $\mathcal{L}[\sinh\omega t] = \frac{\omega}{s^2-\omega^2}$
  - $\mathcal{L}[t^n] = \frac{n!}{s^{n+1}}$
  - $\mathcal{L}[f'(t)] = sF(s) - f(0)$ $\mathcal{L}[f''(t)] = s^2F(s) - sf(0) - f'(0)$
  - $\mathcal{L}[f(t-a)H(t-a)] = F(s)e^{-as}$ t-shift
  - $\mathcal{L}[f(t)] = \frac{1}{1-e^{-Ts}}\int_0^T f(t)e^{-st}dt$ given $f(t)$ is periodic with period T.
  - $\mathcal{L}[\delta(t)] = 1$